EE 330
Lecture 24

- Small Signal Analysis

Exam Schedule

Exam 2 will be given on Friday March 11

Exam 3 will be given on Friday April 15

Photo courtesy of the director of the National Institute of Health (NIH)

As a courtesy to fellow classmates, TAs, and the instructor

Wearing of masks during lectures and in the laboratories for this course would be appreciated irrespective of vaccination status

Amplification with Transistors

From Wikipedia: (Oct. 2019)

> An **amplifier**, **electronic amplifier** or (informally) **amp** is an electronic device that can increase the power of a signal (a time-varying voltage or current).

What is the "power" of a signal?

Can an amplifier make decisions?

Does Wikipedia have such a basic concept right?

Operating Point of Electronic Circuits

Often interested in circuits where a small signal input is to be amplified (e.g. V_M in previous slide is small)

The electrical port variables where the small signals goes to 0 are termed the Operating Points, the Bias Points, the Quiescent Points, or simply the Q-Points

By setting the small signal inputs to 0, it means replacing small voltage inputs with short circuits and small current inputs with open circuits

When analyzing small-signal amplifiers, it is necessary to obtain the Q-point

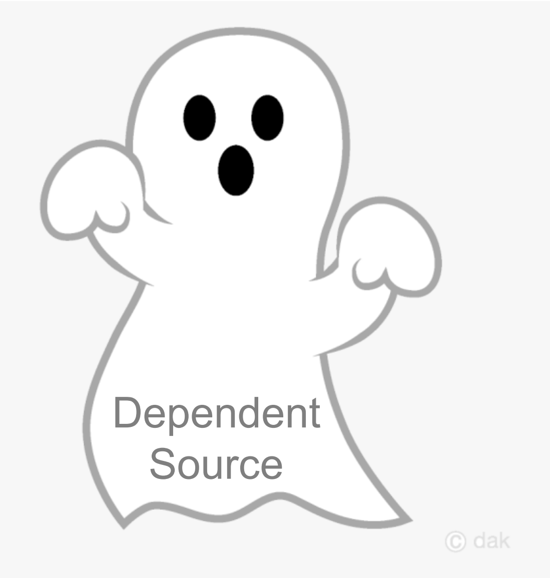
When designing small-signal amplifiers, establishing of the desired Q-point is termed "biasing"

- Capacitors become open circuits (and inductors short circuits) when determining Q-points
- Simplified dc models of the MOSFET (saturation region) or BJT (forward active region) are usually adequate for determining the Q-point in practical amplifier circuits
- DC voltage and current sources remain when determining Q-points
- Small-signal voltage and current sources are set to 0 when determining Q-points

Dependent Sources

What is a dependent source?

Will you suddenly find dependent sources after you graduate ?



Do dependent sources really exist ?

Why do we place so much emphasis on dependent sources in EE 201?

Amplification with Transistors

Often the voltage gain of an amplifier is larger than 1

$$V_{OUT} = A_V V_{IN}$$

Often (but not always) the power dissipated by R_L is larger than the power supplied by V_{IN}

An amplifier can be thought of as a dependent source that was discussed in EE 201

Input and output variables can be either V or I or mixed

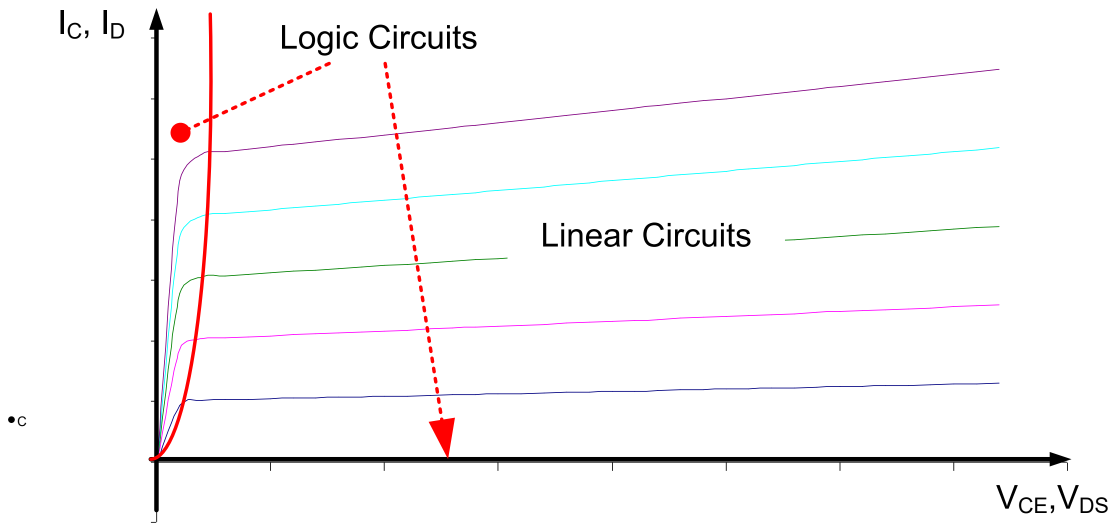
Amplifier

~~From Wikipedia: (March 2022)~~

~~An **amplifier**, **electronic amplifier** or (informally) **amp** is an electronic device that can increase the power of a signal (a time-varying voltage or current).~~

An amplifier is another name for any for the four basic dependent sources that are discussed in basic circuits textbooks.

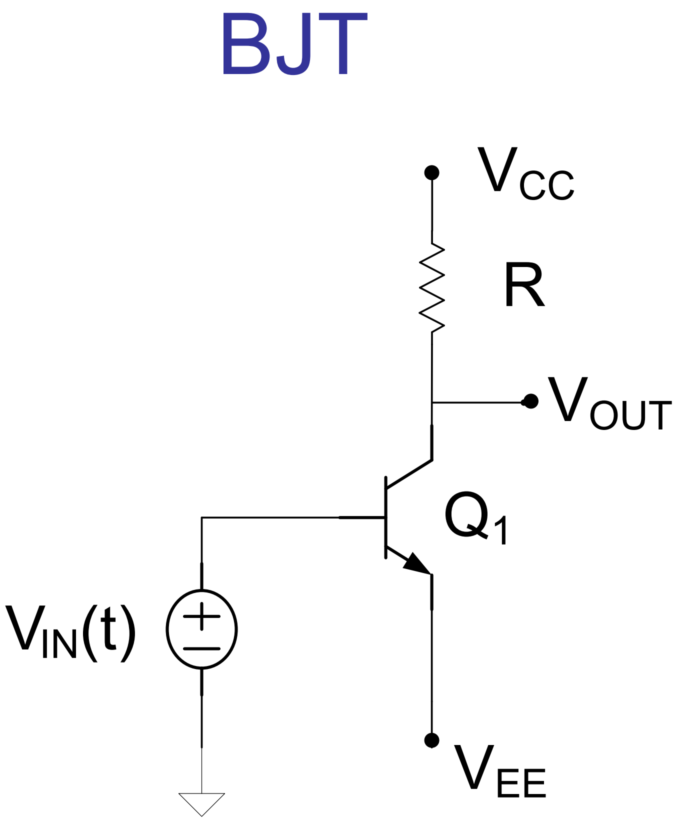
Applications of Devices as Amplifiers

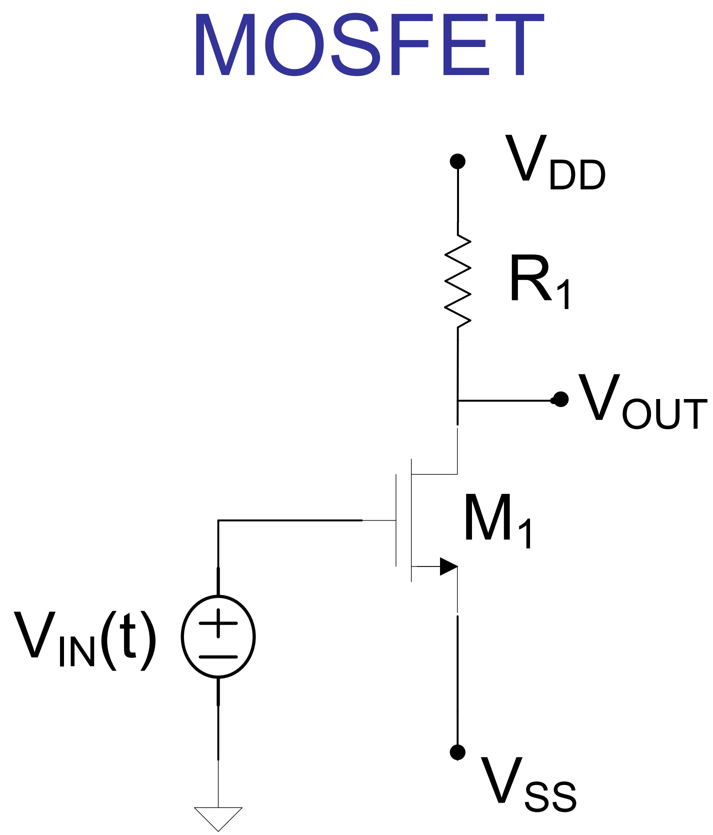


Typical Regions of Operation by Circuit Function

	MOS	Bipolar
Logic Circuits	Triode and Cutoff	Saturation and Cutoff
Linear Circuits	Saturation	Forward Active

Consider the following MOSFET and BJT Circuits





Assume BJT operating in FA region, MOSFET operating in Saturation

Assume same quiescent output voltage and same resistor R_1

Note architecture is same for BJT and MOSFET circuits

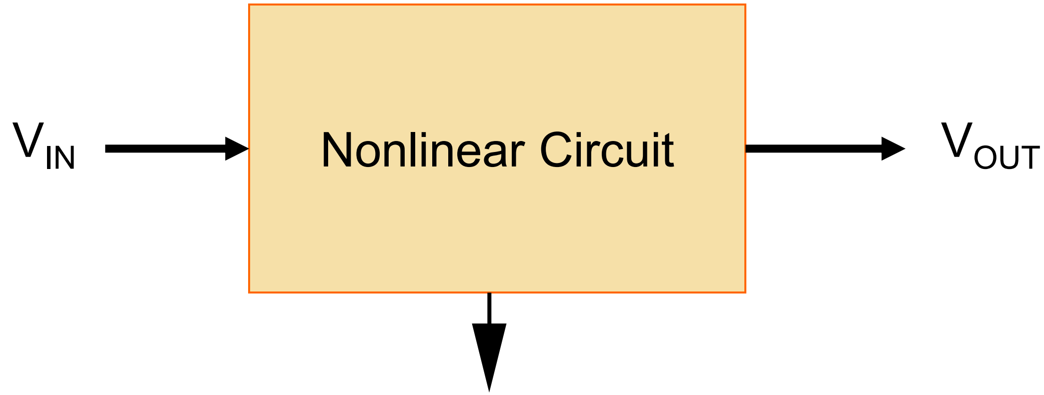
One of the most widely used amplifier architectures

Review from Last Lecture

Small signal operation of nonlinear circuits

$V_{IN}=V_M\sin\omega t$

V_M is small



Practical methods of analyzing and designing circuits that operate with small signal inputs are really important

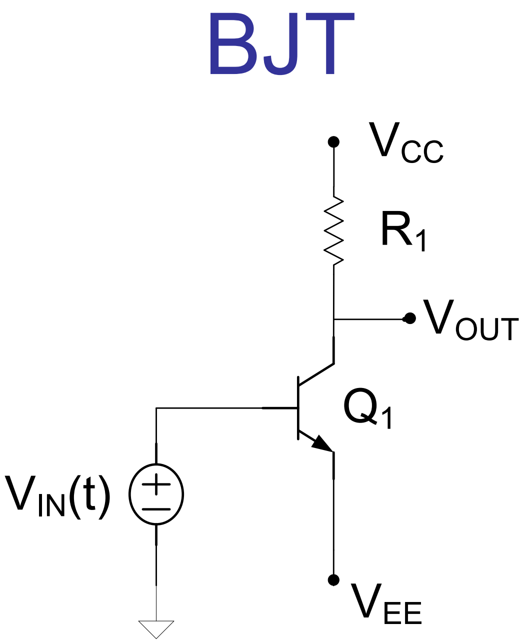
Two key questions:

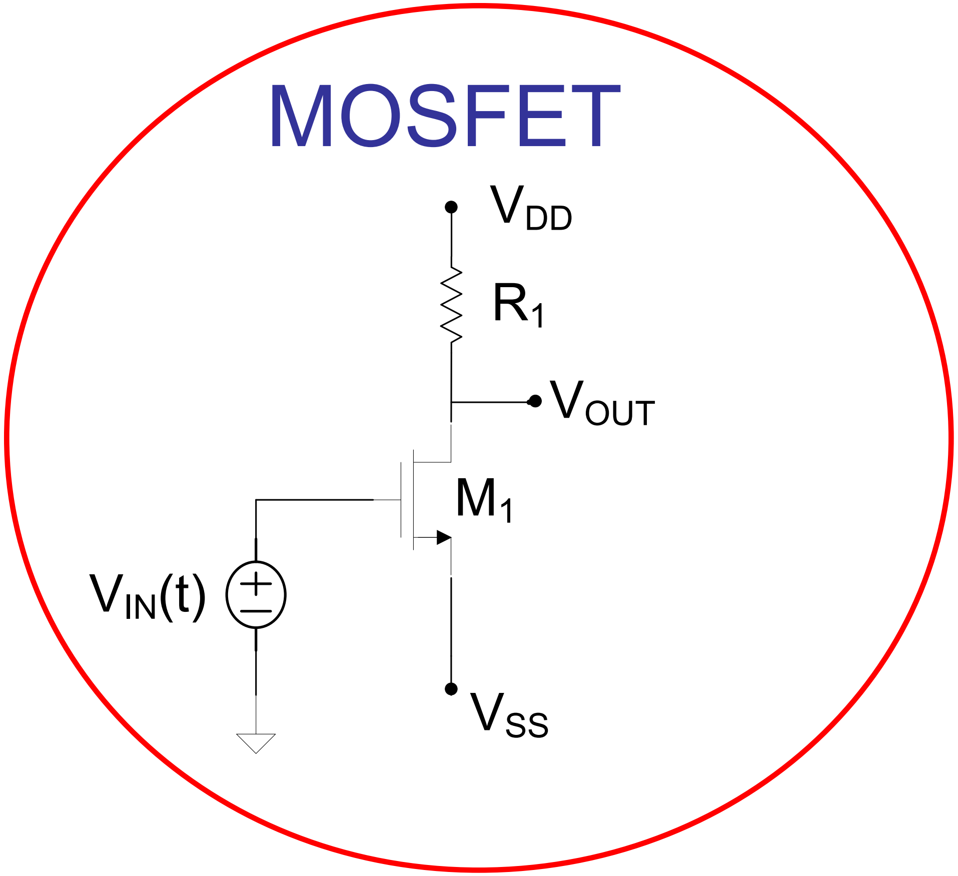
How small must the input signals be to obtain locally-linear operation of a nonlinear circuit?

How can these locally-linear (alt small signal) circuits be analyzed and designed?

Review from Last Lecture

Consider the following MOSFET and BJT Circuits





One of the most widely used amplifier architectures

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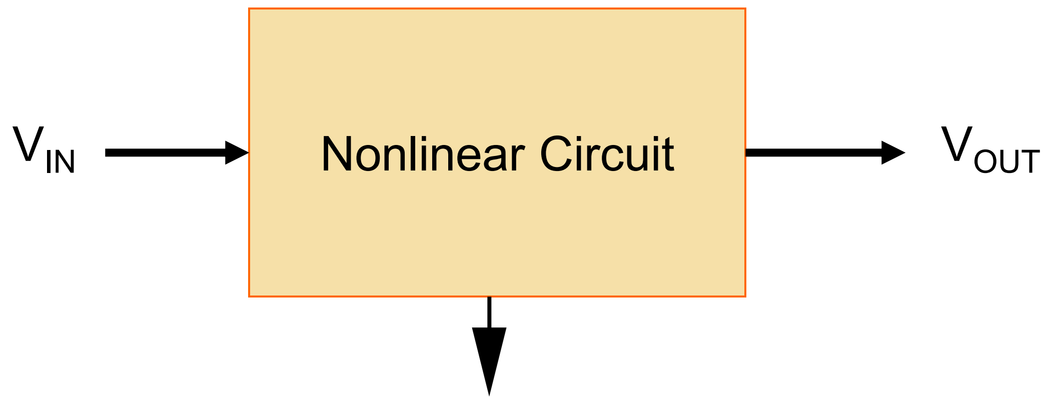
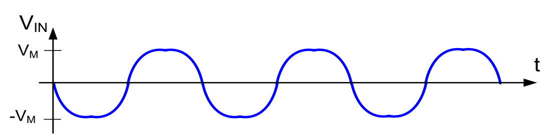
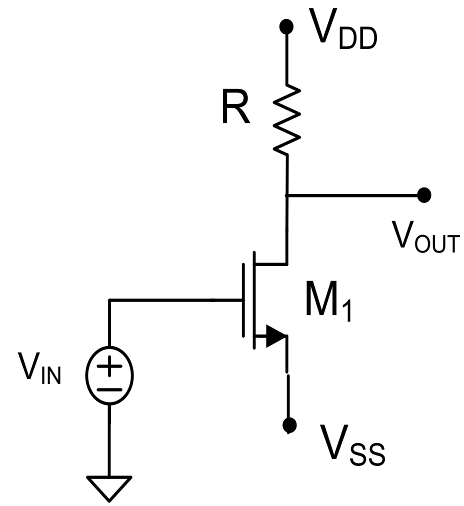


Example of circuit that is widely used in locally-linear mode of operation



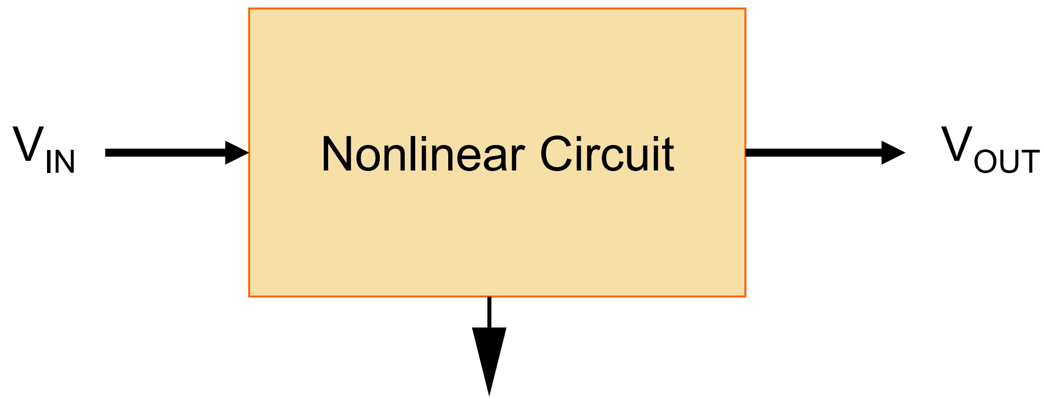


Two methods of analyzing locally-linear circuits will be considered, one of these is by far the most practical

Small signal operation of nonlinear circuits

$V_{IN}=V_M\sin\omega t$

V_M is small

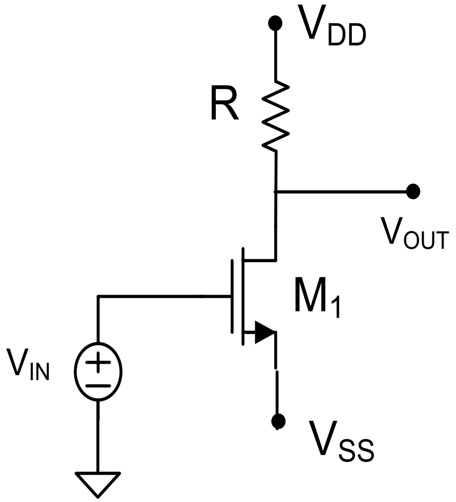


Two methods of analyzing locally-linear circuits for small-signal excitaions will be considered, one of these is by far the most practical

1. Analysis using nonlinear models

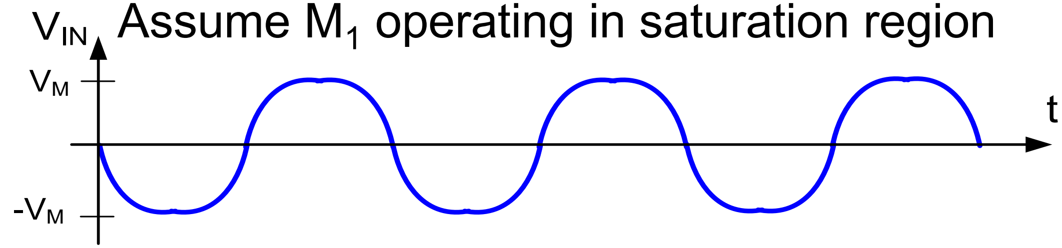
2. Small signal analysis using locally-linearized models

Small signal analysis using nonlinear models



By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region



$V_{IN}=V_M \sin\omega t$

V_M is small

Termed Load Line

$$V_{OUT}=V_{DD}-I_D R$$

$$I_D=\frac{\mu C_{OX}W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2$$

$$\mathbf{I_{DQ}}=\frac{\mu C_{OX}W}{2L}\left(V_{SS}+V_T\right)^2$$

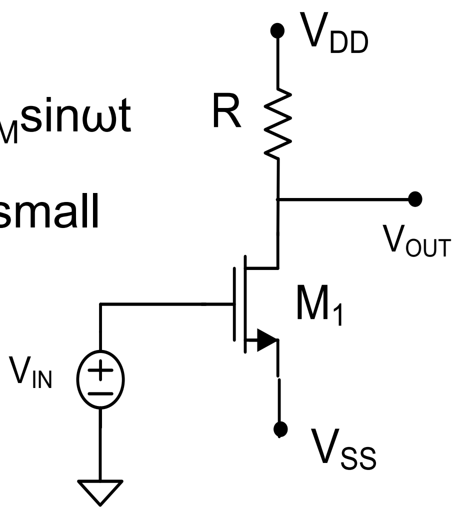
$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_{IN}-V_{SS}-V_T\right)^2 R$$

$$V_{OUT}=V_{DD}-\frac{\mu C_{OX}W}{2L}\left(V_M\sin\omega t-\left[V_{SS}+V_T\right]\right)^2 R$$

Small signal analysis example

$V_{IN}=V_M \sin\omega t$

V_M is small



$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}\left(V_M \sin\omega t - [V_{SS}+V_T]\right)^2 R$$

Note relationship between input and output not linear !

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS}+V_T]^2\left(1 - \frac{V_M \sin\omega t}{[V_{SS}+V_T]}\right)^2 R$$

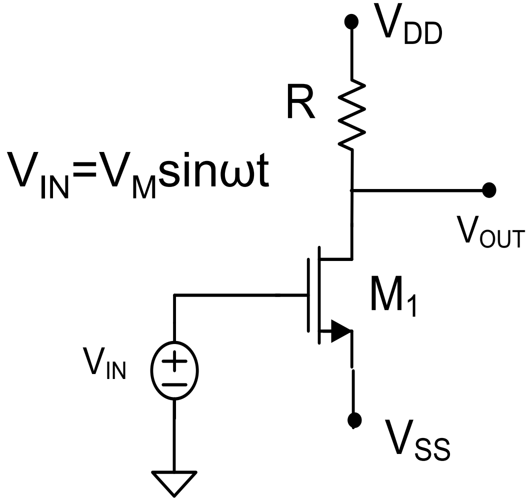
Recall that if x is small $(1+x)^2 \cong 1+2x$

$$V_{OUT} \cong V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS}+V_T]^2\left(1 - \frac{2V_M \sin\omega t}{[V_{SS}+V_T]}\right) R$$

$$V_{OUT} \cong \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS}+V_T]^2 R\right\} + \frac{\mu C_{OX} W}{2L}[V_{SS}+V_T]^2\left(\frac{2V_M \sin\omega t}{[V_{SS}+V_T]}\right) R$$

$$V_{OUT} \cong \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}[V_{SS}+V_T]^2 R\right\} + \left\{\frac{\mu C_{OX} W}{L}[V_{SS}+V_T] R\right\} V_M \sin\omega t$$

Small signal analysis example

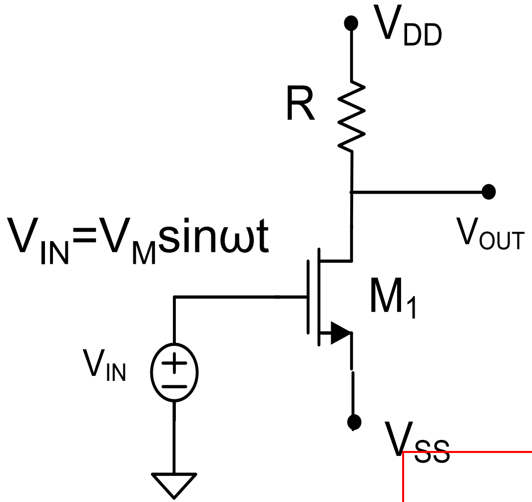


By selecting appropriate value of V_{SS}, M_1 will operate in the saturation region

Assume M_1 operating in saturation region

$$V_{OUT} \cong \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS}+V_T\right]^2 R\right\} + \left\{\frac{\mu C_{OX} W}{L}\left[V_{SS}+V_T\right] R\right\} V_M \sin\omega t$$

Small signal analysis example



Assume M_1 operating in saturation region

$$V_{OUT} \cong \underbrace{\left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS}+V_T\right]^2 R\right\}}_{\text{Quiescent Output}} + \underbrace{\left\{\frac{\mu C_{OX} W}{L}\left[V_{SS}+V_T\right] R\right\}}_{\text{ss Voltage Gain}} V_M \sin\omega t$$

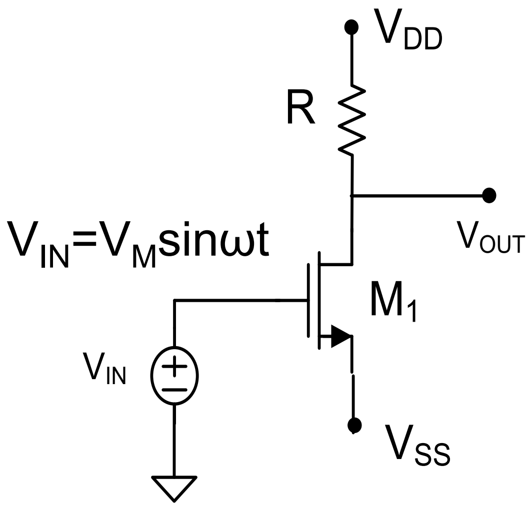
$$A_V = \frac{\mu C_{OX} W}{L}\left[V_{SS}+V_T\right] R$$

$$V_{OUTQ} = \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS}+V_T\right]^2 R\right\}$$

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

Note the ss voltage gain is negative since $V_{SS}+V_T<0$!

Small signal analysis example



Assume M_1 operating in saturation region

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

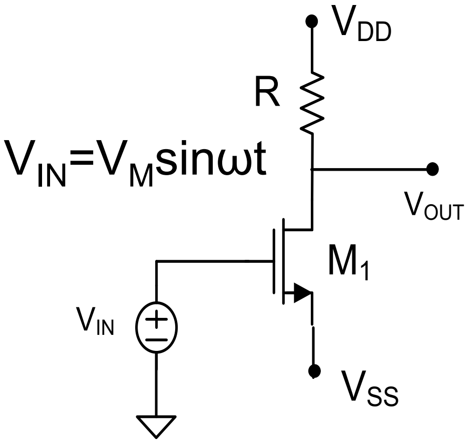
$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right]R$$

$$V_{OUTQ} = \left\{V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{SS} + V_T\right]^2 R\right\}$$

But – this expression gives little insight into how large the gain is !

And the analysis for even this very simple circuit was messy!

Small signal analysis example



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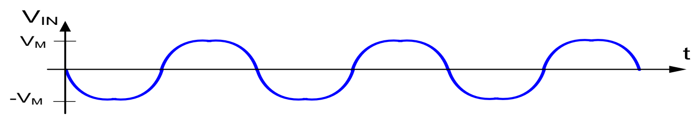
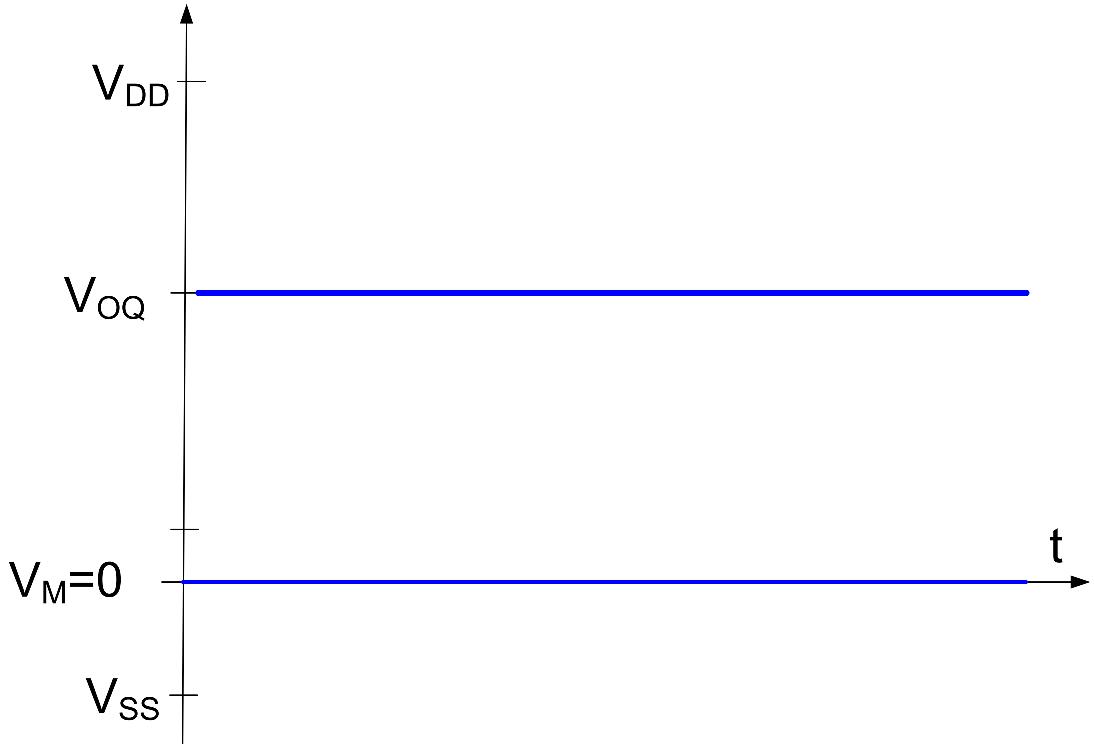
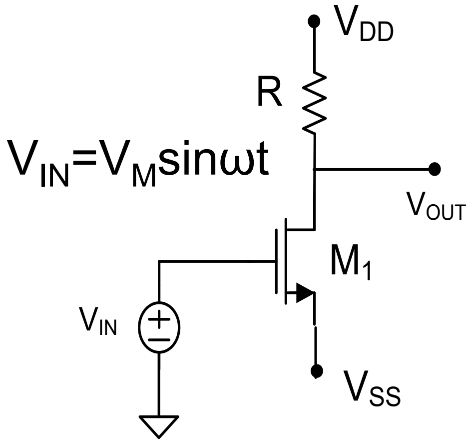


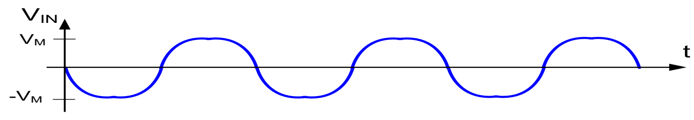


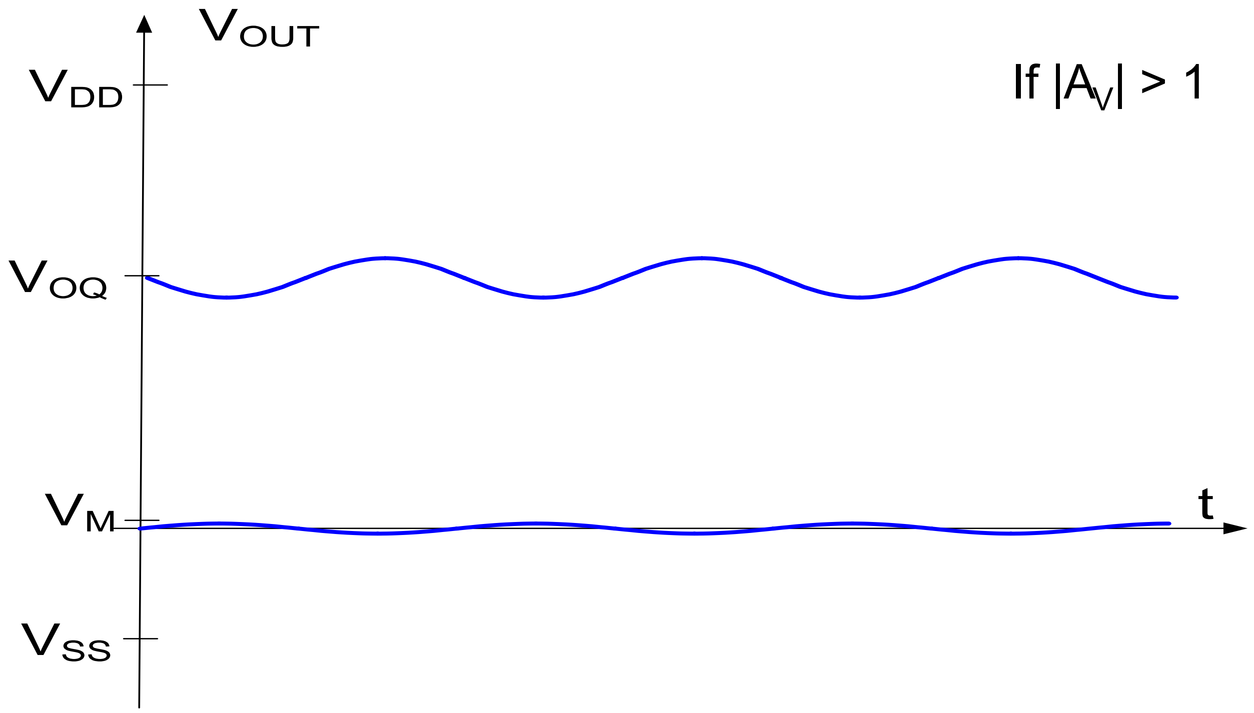
Small signal analysis example



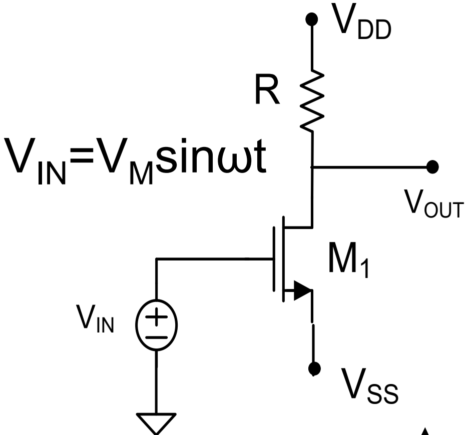
$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$





Small signal analysis example



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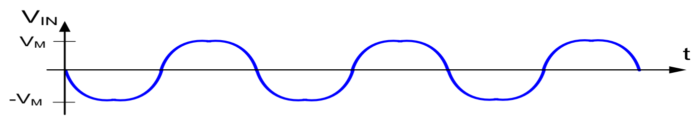
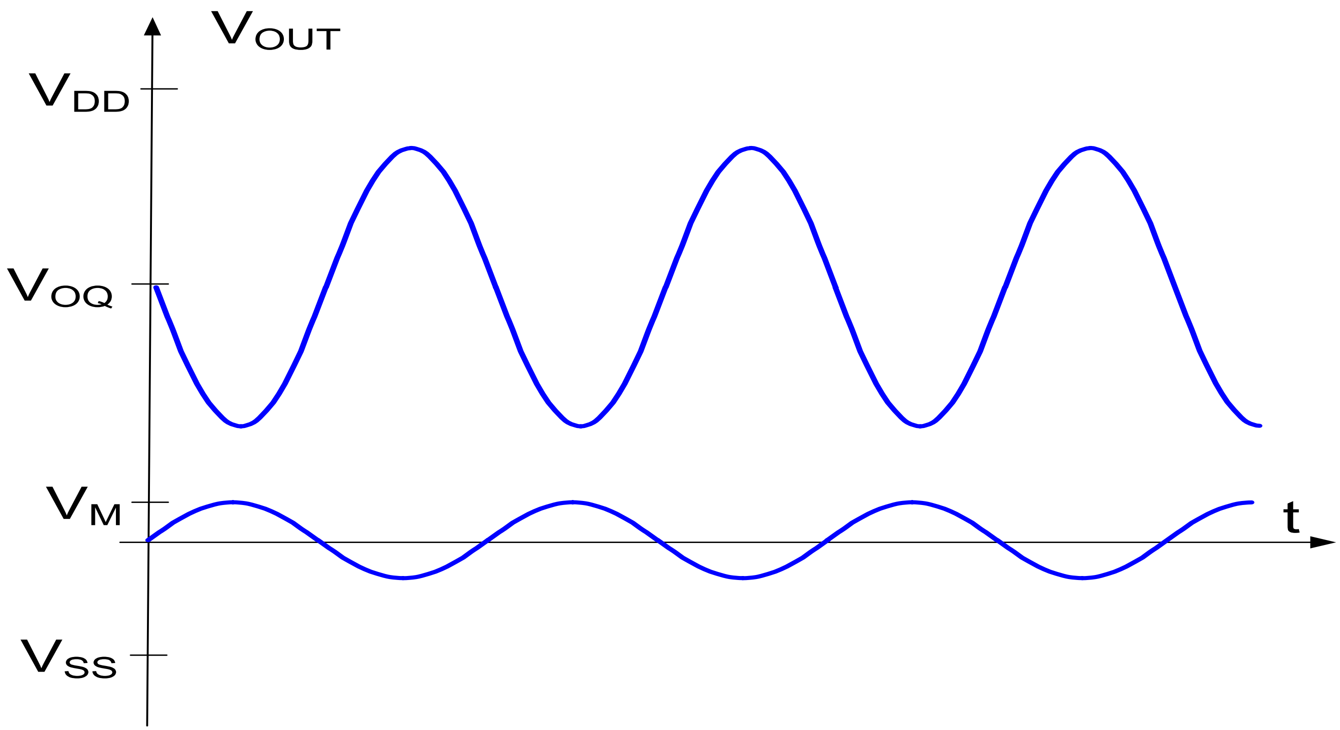
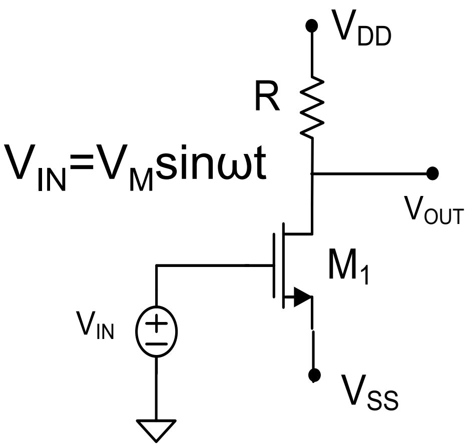
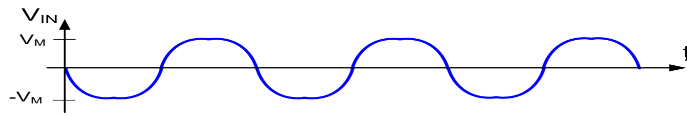




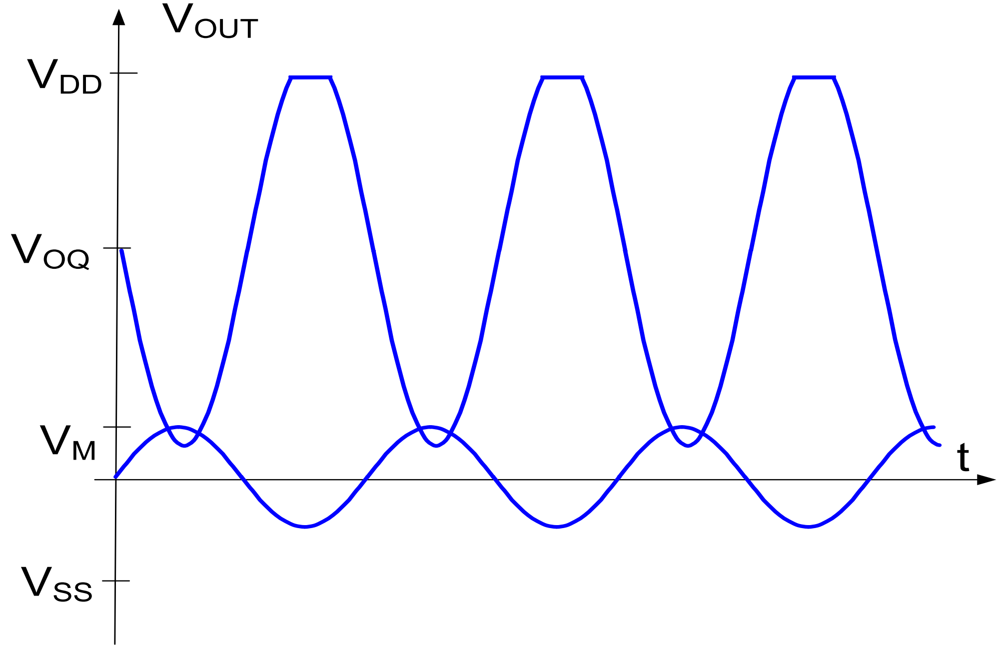
Small signal analysis example

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$



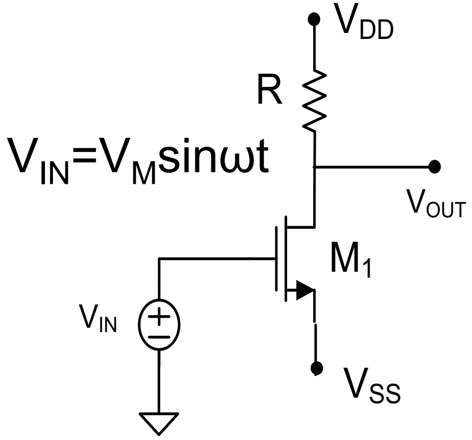


$$A_v = \frac{\mu C_{OX} W}{L}\left[V_{SS} + V_T\right] R$$



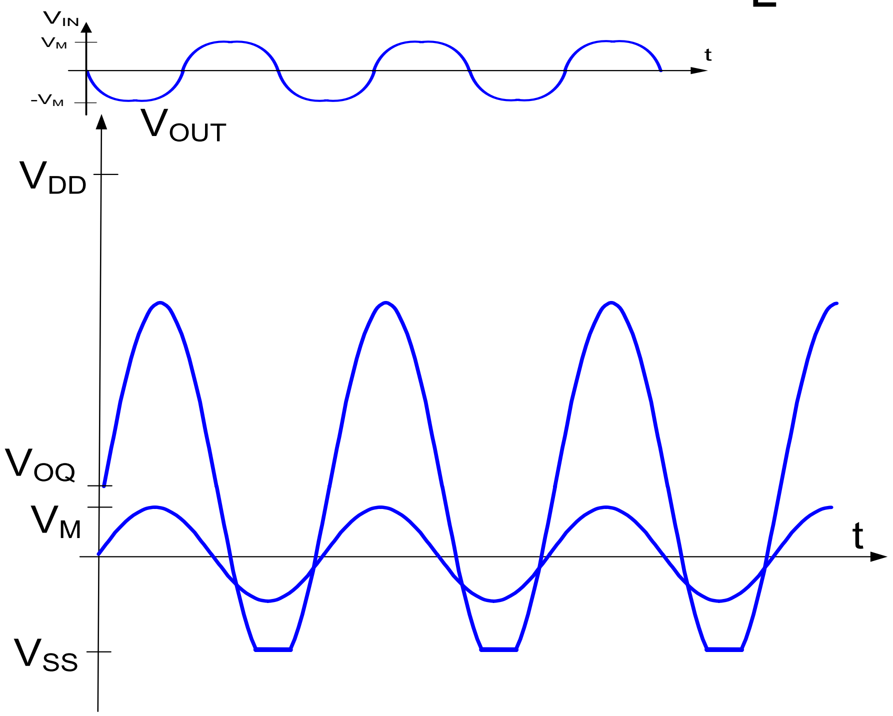
Serious Distortion occurs if signal is too large or Q-point non-optimal
Here “clipping” occurs for high V_{OUT}

Small signal analysis example



$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

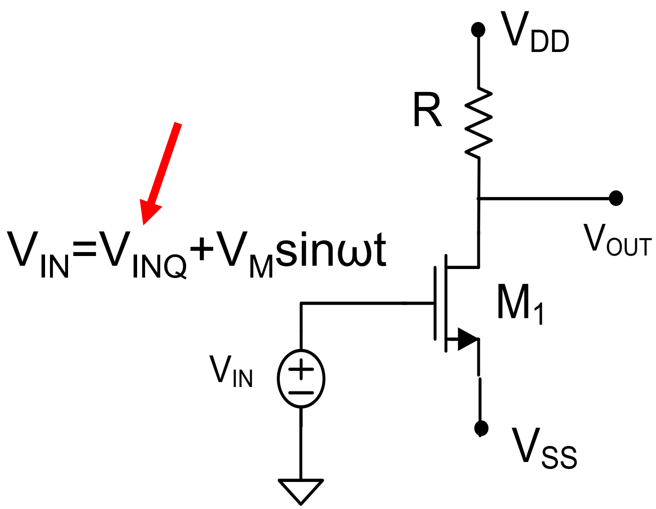
$$A_v = \frac{\mu C_{ox} W}{L}\left[V_{ss} + V_T\right] R$$



Serious Distortion occurs if signal is too large or Q-point non-optimal

Here "clipping" occurs for low V_{OUT}

Small signal analysis example



Assume M_1 operating in saturation region
When $V_{IN}=V_{INQ}$, the Q-point solution:

$$I_{DQ} = \frac{\mu C_{OX} W}{2L}\left(V_{INQ}-V_{SS}-V_T\right)^2$$

$$V_{OUTQ}=V_{DD} - \frac{\mu C_{OX} W}{2L}\left[V_{INQ}-V_{SS}-V_T\right]^2 R$$

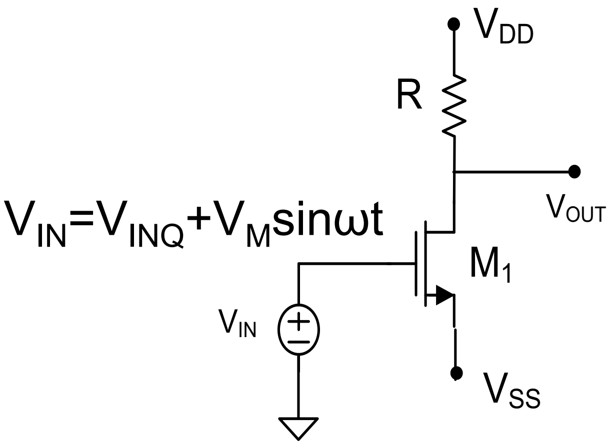
Near the Q-point, small signals have linear relationship:

$$V_{OUTsmall} = V_{OUT} - V_{OUTQ} \cong A_V \cdot (V_{IN} - V_{INQ}) = A_V V_M \sin\omega t$$

$$V_{OUTsmall} \cong A_V V_{INsmall}$$

$$A_V \cong -\frac{\mu C_{OX} W}{L}\left[V_{INQ}-V_{SS}-V_T\right]R$$

Small signal analysis example



$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_V \cong -\frac{\mu C_{OX} W}{L}\left[V_{INQ}-V_{SS}-V_T\right]R$$

$$V_{OUTQ}=V_{DD}-\frac{\mu C_{OX} W}{2L}\left[V_{INQ}-V_{SS}-V_T\right]^2 R$$

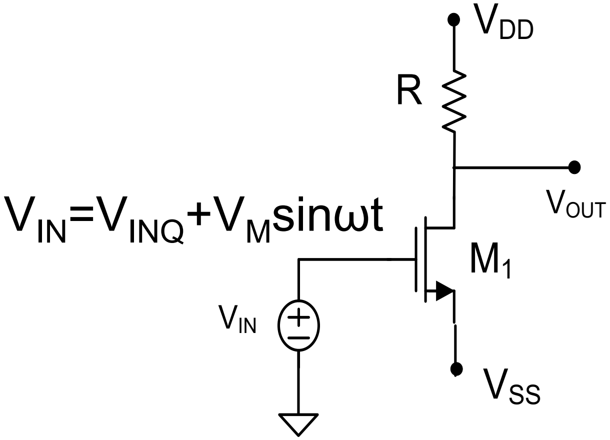
But – this expression gives little insight into how large the gain is !

Can the gain be made arbitrarily large by simply making R large?

Observe increasing R with W,L, and V_{SS} fixed will change Q-point

Difficult to answer this question with the information provided !

Small signal analysis example



$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$A_V \cong -\frac{\mu C_{OX} W}{L}\left[V_{INQ}-V_{SS}-V_T\right]R$$

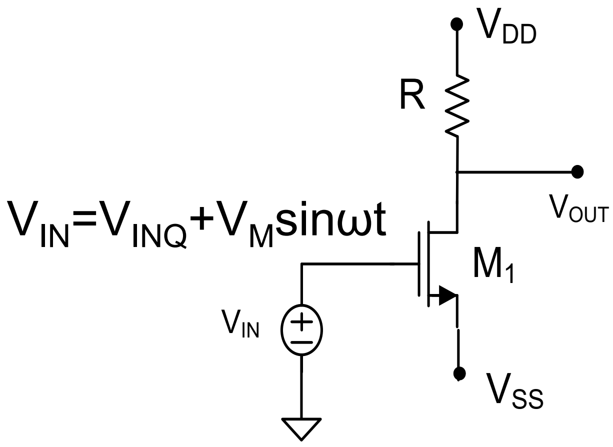
But recall:

$$I_{DQ} = \frac{\mu C_{OX} W}{2L}\left(V_{INQ}-V_{SS}-V_T\right)^2$$

Thus, substituting from the expression for I_{DQ} we obtain

$$A_v = -\frac{2I_{DQ}R}{\left[V_{INQ}-V_{SS}-V_T\right]} = -\frac{2I_{DQ}R}{\left[V_{GSQ}-V_T\right]}$$

Small signal analysis example



$$A_v = -\frac{2I_{DQ}R}{[V_{GSQ}-V_T]}$$

- Small signal voltage gain is twice the Quiescent voltage across R divided by V_{GSQ} - V_T
- Making $I_{DQ}R$ too big or too small will limit signal swing (cause M_1 to leave saturation region)
- Can make $|A_V|$ large by making V_{GSQ} - V_T small
- A_V increases proportionally to the power dissipation (from supply) for fixed V_{GSQ}

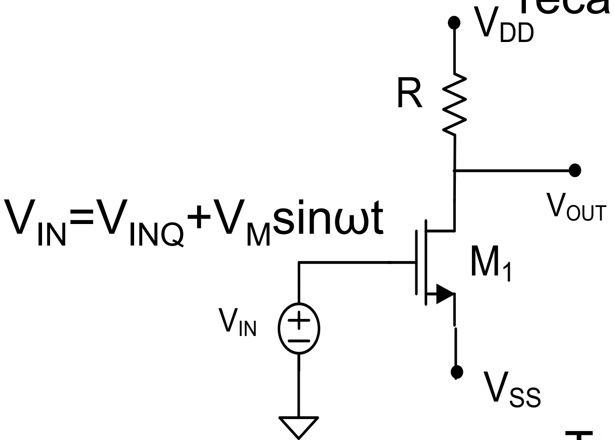
- This analysis which required linearization of a nonlinear output voltage is quite tedious.

- This approach becomes unwieldy for even slightly more complicated circuits

- A much easier approach based upon the development of small signal models will provide the same results, provide more insight into both analysis and design, and result in a dramatic reduction in computational requirements

Small signal analysis example

(Consider what was neglected in the previous analysis)



recall

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

However, there are invariably small errors in this analsis

$$V_{OUT} = V_{OUTQ} + A_V V_M \sin\omega t + \varepsilon(t)$$

To see the effects of the approximations consider again

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(V_M \sin(\omega t) + \left[V_{GSQ} - V_T\right]\right)^2$$

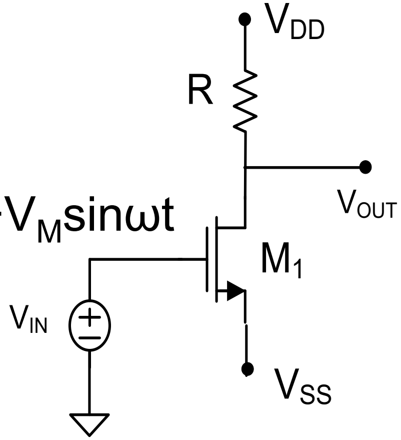
$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(V_M^2 \sin(\omega t)^2 + 2\left[V_{GSQ} - V_T\right]V_M \sin\omega t + \left[V_{GSQ} - V_T\right]^2\right)$$

$$V_{OUT} = V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(V_M^2\left[\frac{1-\cos 2\omega t}{2}\right] + 2\left[V_{GSQ} - V_T\right]V_M \sin\omega t + \left[V_{GSQ} - V_T\right]^2\right)$$

$$V_{OUT} = \left\{V_{DD} - \frac{\mu C_{OX} RW}{2L}\left(\frac{V_M^2}{2} + \left[V_{GSQ} - V_T\right]^2\right)\right\} - \left\{\left(\frac{\mu C_{OX} W}{L}\left[V_{GST} - V_T\right]R\right)V_M \sin\omega t\right\} + \boxed{\left\{\left(\frac{\mu C_{OX} RW}{4L} V_M^2\right)\cos 2\omega t\right\}}$$

Note presence of second harmonic distortion term !

Small signal analysis example



$V_{IN}=V_{INQ}+V_M \sin\omega t$

Nonlinear distortion term

$$V_{OUT} \cong V_{OUTQ} + A_V V_M \sin\omega t$$

$$V_{OUT} = V_{OUTQ} + A_V V_M \sin\omega t + \varepsilon(t)$$

$$V_{OUT}=\left\{V_{DD}-\frac{\mu C_{OX}RW}{2L}\left(\frac{V_M^2}{2}+\left[V_{GSQ}\text{-}V_T\right]^2\right)\right\}-\left\{\left(\frac{\mu C_{OX}W}{L}\left[V_{GSQ}\text{-}V_T\right]R\right)V_M\sin\omega t\right\}+\left\{\left(\frac{\mu C_{OX}RW}{4L}V_M^2\right)\cos 2\omega t\right\}$$

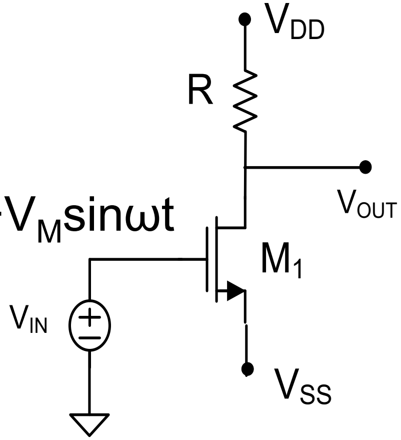
$$V_{OUTDC}=V_{OUTQ}-\frac{\mu C_{OX}RW}{4L}V_M^2$$

$$A_V=-\frac{\mu C_{OX}W}{L}\left[V_{GSQ}\text{-}V_T\right]R$$

$$A_2=\frac{\mu C_{OX}RW}{4L}V_M$$

$$V_{OUT}=V_{OUTDC}+\left\{A_V V_M\sin\omega t\right\}+\left\{A_2 V_M\cos 2\omega t\right\}$$

Small signal analysis example



$V_{IN}=V_{INQ}+V_M\sin\omega t$

Nonlinear distortion term

$$V_{OUT}=V_{OUTDC}+\{A_V V_M \sin\omega t\}+\{A_2 V_M \cos 2\omega t\}$$

$$A_V=-\frac{\mu C_{OX}W}{L}\left[V_{GSQ}\text{-}V_T\right]R \qquad A_2=\frac{\mu C_{OX}RW}{4L}V_M$$

Total Harmonic Distortion:

Recall, if $x(t)=\sum_{k=0}^{\infty} b_k \sin(k\omega T+\phi_k)$ then $\mathrm{THD}=\frac{\sqrt{\sum_{k=2}^{\infty} b_k^2}}{|b_1|}$

Thus, for this amplifier, as long as M_1 stays in the saturation region

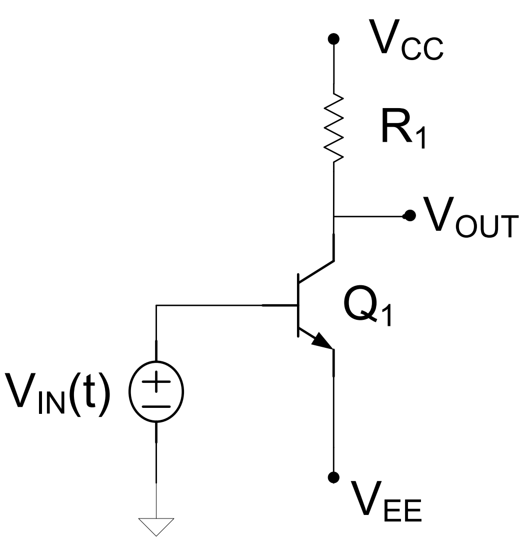
$$\mathrm{THD}=\frac{\sqrt{(A_2V_M)^2}}{|A_V V_M|}=\frac{A_2}{|A_V|}=\frac{\frac{\mu C_{OX}W}{4L}RV_M}{\frac{\mu C_{OX}W}{L}R(V_{GSQ}\text{-}V_T)}=\frac{V_M}{4(V_{GSQ}\text{-}V_T)}$$

Distortion will be small for $V_M \ll (V_{GSQ}-V_T)$

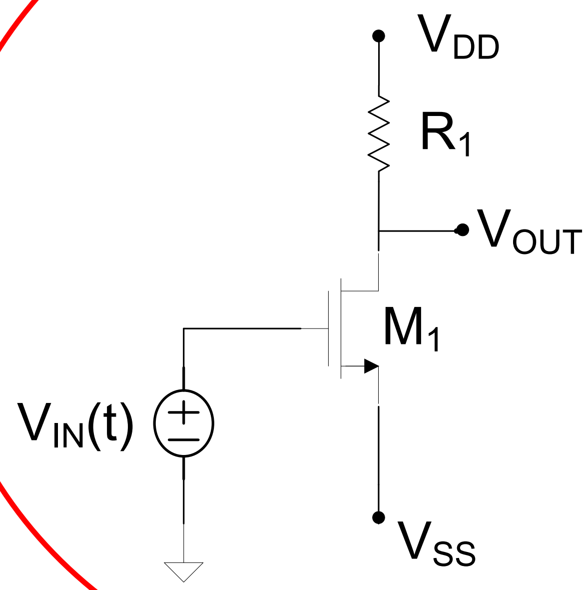
Distortion will be much worse (larger and more harmonic terms) if M_1 leaves saturation region.

Consider the following MOSFET and BJT Circuits

BJT



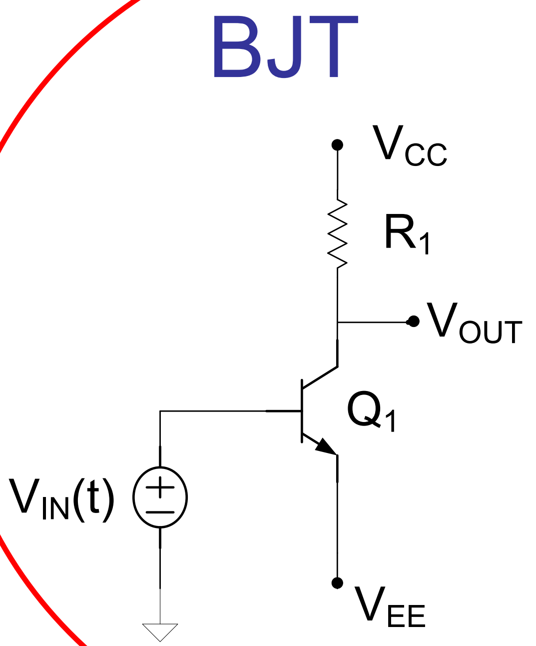
MOSFET

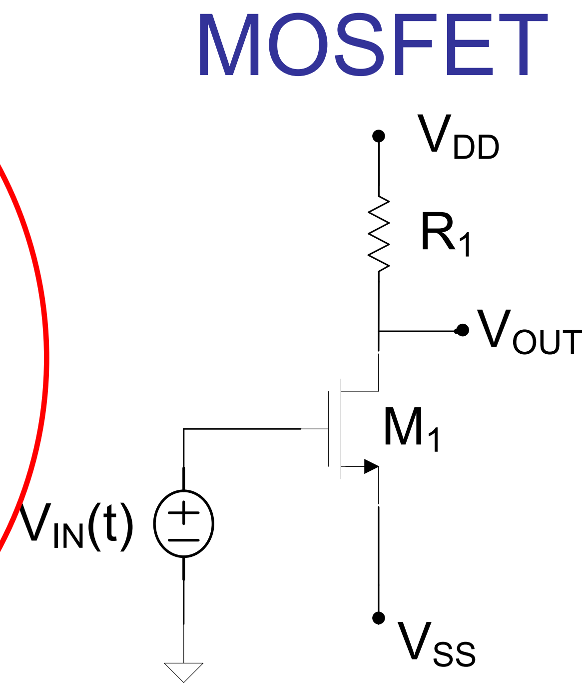


- Analysis was very time consuming
- Issue of operation of circuit was obscured in the details of the analysis

One of the most widely used amplifier architectures

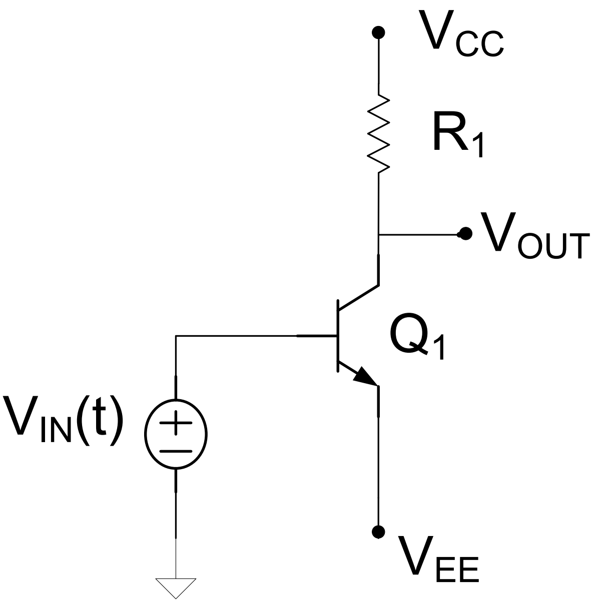
Consider the following MOSFET and BJT Circuits





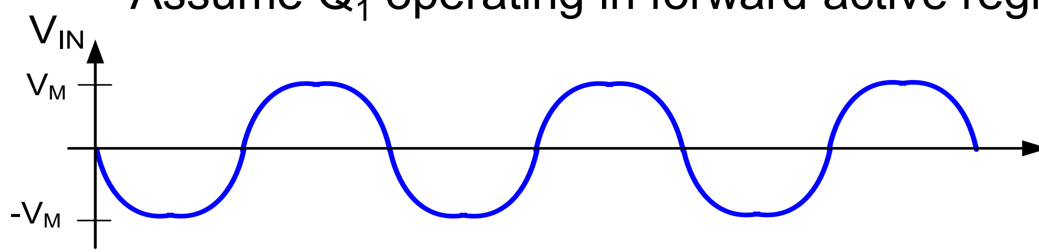
One of the most widely used amplifier architectures

Small signal analysis using nonlinear models



By selecting appropriate value of V_{SS}, M_1 will operate in the forward active region

Assume Q_1 operating in forward active region



$V_{IN}=V_{INQ}+V_M \sin\omega t$

V_M is small

$$V_{OUT}=V_{CC}-I_C R_1$$

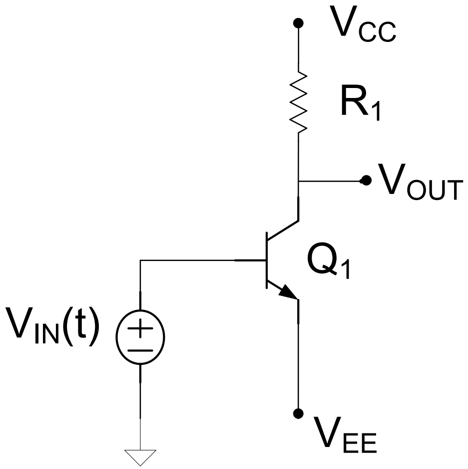
$$I_C = J_S A_E e^{\frac{V_{IN}-V_{EE}}{V_t}}$$

$$\mathbf{I_{CQ} = J_S A_E e^{\frac{V_{INQ}-V_{EE}}{V_t}} = J_S A_E e^{\frac{V_{beQ}}{V_t}}}$$

$$V_{OUTQ} = V_{CC} - J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}}$$

$$V_{OUT} = V_{CC} - J_S A_E R_1 e^{\frac{V_M \sin(\omega t)+V_{beQ}}{V_t}}$$

Small signal analysis using nonlinear models



$$I_{CQ} = J_S A_E e^{\frac{V_{beQ}}{V_t}}$$

$$V_{OUT} = V_{CC} - J_S A_E R_1 e^{\frac{V_M \sin(\omega t) + V_{beQ}}{V_t}}$$

$$V_{OUT} = V_{CC} - J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}} e^{\frac{V_M \sin(\omega t)}{V_t}}$$

Recall that if x is small $e^{\varepsilon} \cong 1+\varepsilon$ (truncated Taylor's series)

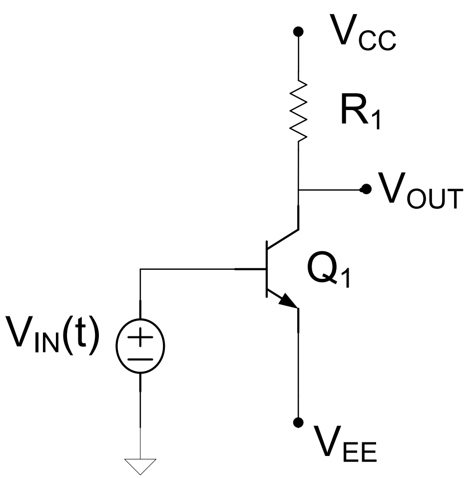
$V_{IN}=V_{INQ}+V_M \sin\omega t$

V_M is small

$$\therefore V_{OUT} \cong V_{CC} - J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}} \left(1 + \frac{V_M \sin(\omega t)}{V_t}\right)$$

$$V_{OUT} \cong \left[V_{CC} - J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}}\right] - \frac{J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}}}{V_t} V_M \sin(\omega t)$$

Small signal analysis using nonlinear models



$$I_{CQ} = J_S A_E e^{\frac{V_{beQ}}{V_t}}$$

$$V_{OUT} \cong \left[V_{CC} - \boxed{J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}}} \right] - \frac{\boxed{J_S A_E R_1 e^{\frac{V_{beQ}}{V_t}}}}{V_t} V_M \sin(\omega t)$$

$V_{IN}=V_{INQ}+V_M\sin\omega t$

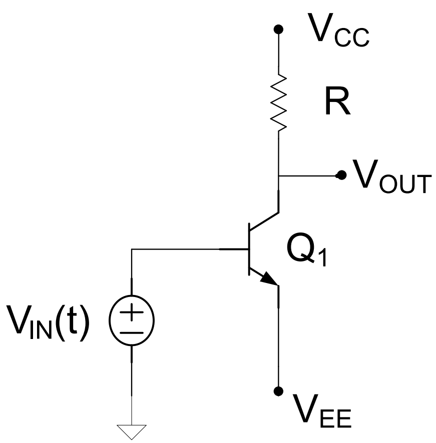
V_M is small

$$V_{OUT} \cong \underbrace{\left[V_{CC} - I_{CQ} R_1 \right]}_{\textit{Quiescent Output}} - \boxed{\left(\frac{I_{CQ} R_1}{V_t} \right)} V_M \sin(\omega t)$$

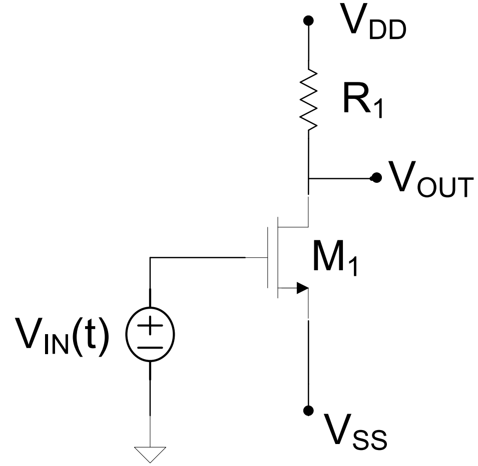
ss Voltage Gain

Comparison of Gains for MOSFET and BJT Circuits

BJT



MOSFET



$$A_{VB} = -\frac{I_{CQ}R}{V_t}$$

$$A_{VM} = -\frac{2I_{DQ}R_1}{V_{GSQ} - V_T}$$

If $I_{DQ}R_1 = I_{CQ}R = 2V$, $V_{GSQ} - V_T = 1V$, $V_t = 25mV$

$$A_{VB} = -\frac{I_{CQ}R}{V_t} = -\frac{2V}{25mV} = -80$$

$$A_{VM} = -\frac{2I_{DQ}R_1}{V_{GSQ} - V_T} = -\frac{4V}{1V} = -4$$

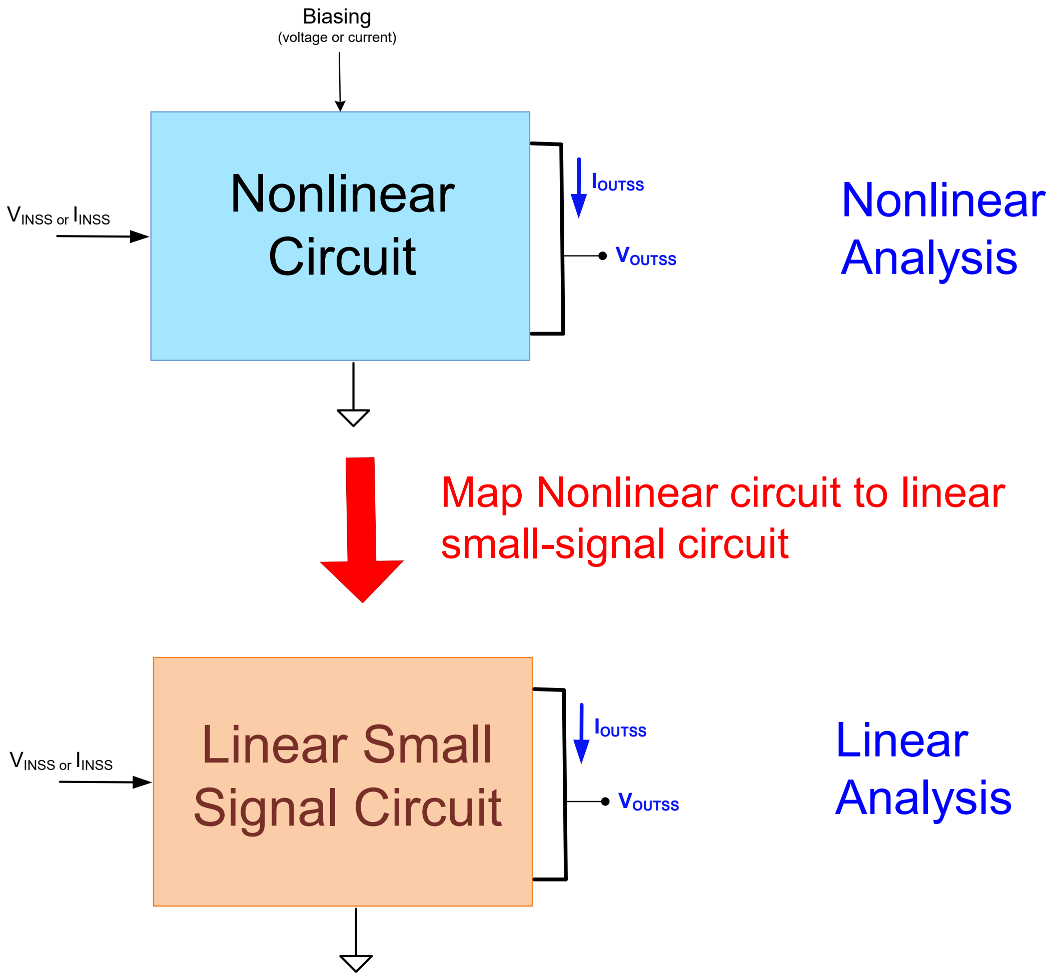
Observe $A_{VB} >> A_{VM}$

Due to exponential-law rather than square-law model

Operation with Small-Signal Inputs

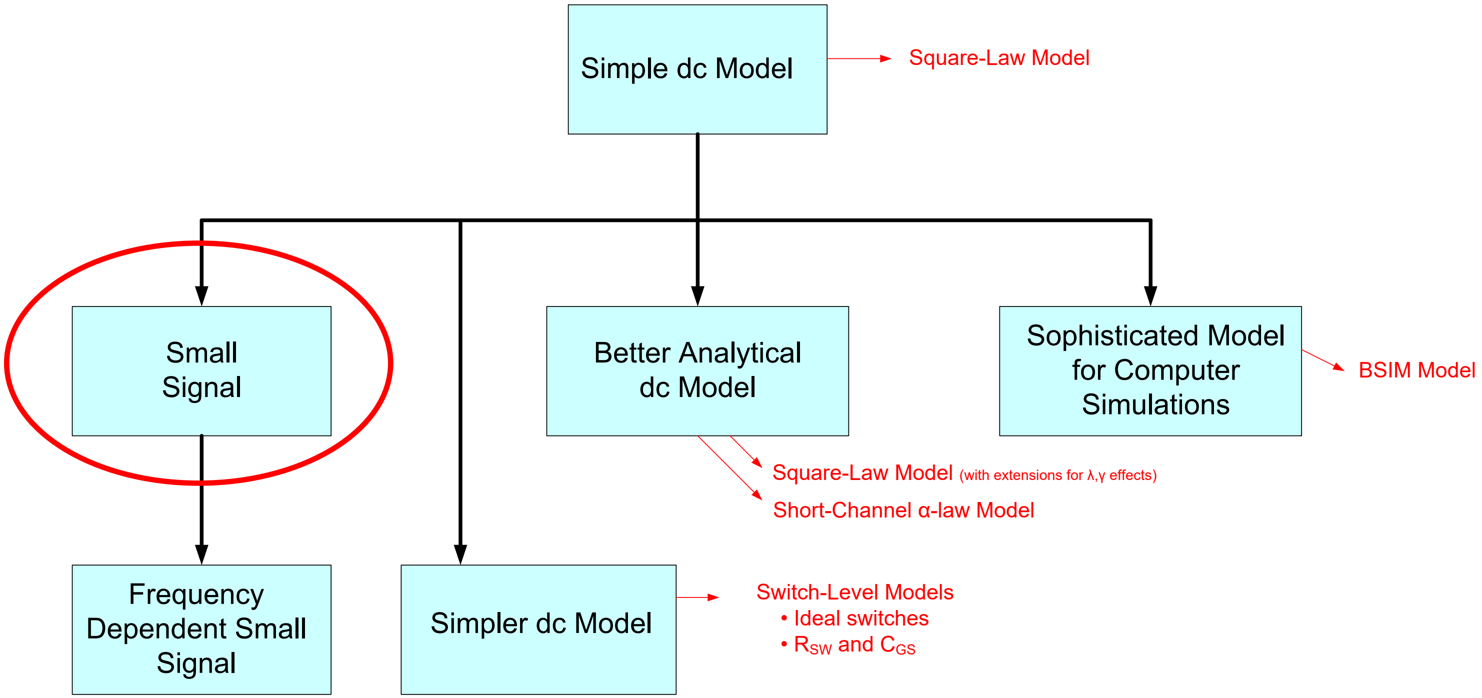
- Analysis procedure for these simple circuits was very tedious
- This approach will be unmanageable for even modestly more complicated circuits
- **Faster analysis method is needed !**

Small-Signal Analysis



- Will commit next several lectures to developing this approach
- Analysis will be MUCH simpler, faster, and provide significantly more insight
- Applicable to many fields of engineering

Small-Signal Analysis



Stay Safe and Stay Healthy !

End of Lecture 24